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branches of the curve can have no point in common except $(0, a)$. Moreover, this point is on the locus only when $\nu a=\lambda$, as may be seen by substituting in equation (1). When this condition is satisfied the two branches are tangent at the point $(0, a)$; for if they should cut each other here they would necessarily have another point in common. Moreover one branch is always entirely within the other.

Now if there is another branch to the locus besides the two already discussed some line passing through the inner of these two ovals cuts this third branch. The line then intersects the *quartic* curve in *five* points; but this is impossible. Hence the entire locus consists of the two ovals above constructed.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

276. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

If $x_1, x_2, x_3, \ldots, x_n$ be unequal, and $f(x)$ be a rational integral function of degree $\not> n-2$, then shall

$$\sum_{r=1}^{r=n-1} \frac{f(x_r)}{(x_r-x_1)(x_r-x_2)\ \ldots\ (x_r-x_n)}=0.$$

No solution of this problem has been received. It is not clear what is intended in the problem. By taking $f(x)=x^2-x+1$ and $x_1=1, x_2=2, x_3=3, x_4=4$, the problem is not true according to our interpretation of it, since infinite addends would be introduced.

In Burnside and Panton's *Theory of Equations*, p. 319, 3rd edition, example 4, slightly changed, reads: If the degree of $\phi(x)$ does not exceed $n-2$, prove that $\sum_{r=1}^{r=n} \frac{\phi(x_r)}{f'(x_r)}=0$, ..., $x_r (r=1, 2, \ldots, n)$ being roots of $f(x)=0$.

Additional information on this problem is desired. ED. F.

277. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

If $\alpha, \beta, \gamma, \delta$ are the roots of the quartic $ax^4+bx^3+cx^2+dx+e=0$, calculate the value of the product of the twelve expressions of the form $(4\alpha-2\beta-\gamma-\delta)$ in terms of H, I, J, the well known functions of the differences of the roots.

Solution by the PROPOSER.

To avoid fractions write the equation in the form $ax^4+4b'x^3+6c'x^2+4d'x+e=0$. See Burnside and Panton's *Theory of Equations*, 4th edition, Vol. I, pp. 121-131.

$H=ac'-b'^2$ $I=ae-4b'd'+3c'^2$, $J=ac'e+2b'c'd'-ad'^2-eb'^2-c'^3$.

$$\left.\begin{array}{l} M_1^2=a^2\theta_1+b'^2-ac'=a^2\theta_1-H \\ M_2^2=a^2\theta_2+b'^2-ac'=a^2\theta_2-H \\ M_3^2=a^2\theta_3+b'^2-ac'=a^2\theta_3-H \end{array}\right\}\ldots(1),$$

where θ_1, θ_2, θ_3 are the roots of the reducing cubic $4a^3\theta^3-Ia\theta+J=0$.

$$\left.\begin{array}{l} a\alpha+b'=-M_1+M_2+M_3 \\ a\beta+b'=M_1-M_2+M_3 \\ a\gamma+b'=M_1+M_2-M_3 \\ a\delta+b'=-M_1-M_2-M_3 \end{array}\right\}\ldots(2).$$

Now $\theta_1+\theta_2+\theta_3=0$, $\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3=-I/4a^2$, $\theta_1\theta_2\theta_3=-J/4a^3$.

The twelve expressions of the form $(4\alpha-2\beta-\gamma-\delta)$ can be easily expressed in terms of M_1, M_2, M_3 from (2), thus

$$\begin{array}{l}
\tfrac{1}{2}a(4\alpha-2\beta-\gamma-\delta)=-[3(M_1-M_2)-2M_3] \\
\tfrac{1}{2}a(4\alpha-\beta-2\gamma-\delta)=-[3(M_1-M_3)-2M_2] \\
\tfrac{1}{2}a(4\alpha-\beta-\gamma-2\delta)=[3(M_2+M_3)-2M_1] \\
\tfrac{1}{2}a(4\beta-2\alpha-\gamma-\delta)=[3(M_1-M_2)+2M_3] \\
\tfrac{1}{2}a(4\beta-\alpha-2\gamma-\delta)=-[3(M_2-M_3)-2M_1] \\
\tfrac{1}{2}a(4\beta-\alpha-\gamma-2\delta)=3(M_1+M_3)-2M_2 \\
\tfrac{1}{2}a(4\gamma-2\alpha-\beta-\delta)=3(M_1-M_3)+2M_2 \\
\tfrac{1}{2}a(4\gamma-\alpha-2\beta-\delta)=3(M_2-M_3)+2M_1 \\
\tfrac{1}{2}a(4\gamma-\alpha-\beta-2\delta)=3(M_1+M_2)-2M_3 \\
\tfrac{1}{2}a(4\delta-2\alpha-\beta-\gamma)=-[3(M_2+M_3)+2M_1] \\
\tfrac{1}{2}a(4\delta-\alpha-2\beta-\gamma)=-[3(M_1+M_3)+2M_2] \\
\tfrac{1}{2}a(4\delta-\alpha-\beta-2\gamma)=-[3(M_1+M_2)+2M_3]
\end{array}$$

These twelve can be reduced to three as follows: $\frac{1}{16}a^4(4\alpha-2\beta-\gamma-\delta)(4\beta-2\alpha-\gamma-\delta)(4\gamma-\alpha-\beta-2\delta)(4\delta-\alpha-\beta-2\gamma)$

$$\begin{array}{l}
=[9(M_1-M_2)^2-4M_3^2][9(M_1+M_2)^2-4M_3^2] \\
=[9M_1^2+9M_2^2-4M_3^2-18M_1M_2][9M_1^2+9M_2^2-4M_3^2+18M_1M_2] \\
=[9a^2\theta_1-9H+9a^2\theta_2-9H-4a^2\theta_3+4H-18M_1M_2]\ [9a^2\theta_1-9H+9a^2\theta_2 \\
\qquad -9H-4a^2\theta_3+4H+18M_1M_2] \\
=[9a^2(\theta_1+\theta_2+\theta_3)-13a^2\theta_3)-14H-18M_1M_2][9a^2(\theta_1+\theta_2+\theta_3) \\
\qquad -13a^2\theta_3-14H+18M_1M_2] \\
=(13a^2\theta_3+14H+18M_1M_2)(13a^2\theta_3+14H-18M_1M_2) \\
=(13a^2\theta_3+14H)^2-324M_1^2M_2^2 \\
=(13a^2\theta_3+14H)^2-324(a^2\theta_1-H)(a^2\theta_2-H)
\end{array}$$

$$=169a^4\theta_3^2+364a^2\theta_3H+196H^2-324a^4\theta_1\theta_2+324a^2\theta_1H+324a^2\theta_2H$$
$$-324H^2$$
$$=169a^4\theta_3^2+40a^2\theta_3+324a^2H(\theta_1+\theta_2+\theta_3)-324a^4\theta_1\theta_2-128H^2$$
$$=169a^4\theta_3^2+40a^2\theta_3+81aJ/\theta_3-128H^2$$
$$=\frac{1}{\theta_3}(169a^4\theta_3^3+40a^2\theta_3^2H+81aJ-128H^2\theta_3)=\frac{1}{\theta_3}(A+81aJ).$$

Similarly,

$$\tfrac{1}{16}a^4(4\alpha-\beta-2\gamma-\delta)(4\beta-\alpha-\gamma-2\delta)(4\gamma-2\alpha-\beta-\delta)(4\delta-\alpha-2\beta-\gamma)$$
$$=\frac{1}{\theta_2}(169a^4\theta_2^3+40a^2\theta_2^2H+81aJ-128H^2\theta_2)=\frac{1}{\theta_2}(B+81aJ).$$
$$\tfrac{1}{16}a^4(4\alpha-\beta-\gamma-2\delta)(4\beta-\alpha-2\gamma-\delta)(4\gamma-\alpha-2\beta-\delta)(4\delta-2\alpha-\beta-\gamma)$$
$$=\frac{1}{\theta_1}(169a^4\theta_1^3+40a^2\theta_1^2H+81aJ-128H^2\theta_1)=\frac{1}{\theta_1}(C+81aJ).$$

Let Π=the product of the twelve factors.

$$\therefore\ \frac{a^{12}\Pi}{4096}=\frac{1}{\theta_1\theta_2\theta_3}[531441a^3J^3+6561a^2J^2(A+B+C)+81aJ(AB+AC$$
$$+BC)+ABC]$$
$$=-\frac{4a^3}{J}[531441a^3J^3+6561a^2J^2(A+B+C)+81aJ(AB+AC+BC)+ABC].$$

$$A+B+C=169a^4(\theta_1^3+\theta_2^3+\theta_3^3)+40a^2H(\theta_1^2+\theta_2^2+\theta_3^2)-128H^2(\theta_1+\theta_2+\theta_3).$$

$$AB+AC+BC=28561a^8(\theta_1^3\theta_2^3+\theta_1^3\theta_3^3+\theta_2^3\theta_3^3)+1600a^4H^2(\theta_1^2\theta_2^2+\theta_1^2\theta_3^2$$
$$+\theta_2^2\theta_3^2)+16384H^4(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)+6760a^6H(\theta_1^2\theta_2^3+\theta_1^3\theta_2^2+\theta_1^2\theta_3^3$$
$$+\theta_1^3\theta_3^2+\theta_2^2\theta_3^3+\theta_2^3\theta_3^2)-21632a^4H^2(\theta_1^3\theta_2+\theta_1\theta_2^3+\theta_1^3\theta_3+\theta_1\theta_3^3+\theta_2^3\theta_3$$
$$+\theta_2\theta_3^3)-5120a^2H^3(\theta_1^2\theta_2+\theta_1\theta_2^2+\theta_1^2\theta_2+\theta_1^2\theta_3+\theta_1\theta_3^2+\theta_2^2\theta_3+\theta_2\theta_3^2).$$

$$ABC=4826809a^{12}\theta_1^3\theta_2^3\theta_3^3+64000a^6H^3\theta_1^2\theta_2^2\theta_3^2-2097152H^6\theta_1\theta_2\theta_3$$
$$+270400a^8H^2\theta_1^2\theta_2^2\theta_3^2(\theta_1+\theta_2+\theta_3)+2768896a^4H^4\theta_1\theta_2\theta_3(\theta_1^2+\theta_2^2+\theta_3^2)$$
$$+1142440a^{10}H\theta_1^2\theta_2^2\theta_3^2(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)-3655808a^8H^2\theta_1\theta_2\theta_3(\theta_1^2\theta_2^2$$
$$+\theta_1^2\theta_3^2+\theta_2^2\theta_3^2)-204800a^4H^4\theta_1\theta_2\theta_3(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)$$
$$+655360a^4H^4\theta_1\theta_2\theta_3(\theta_1+\theta_2+\theta_3)-865280a^6H^3\theta_1\theta_2\theta_3(\theta_1\theta_2^2+\theta_1^2\theta_2$$
$$+\theta_1\theta_3^2+\theta_1^2\theta_3+\theta_2\theta_3^2+\theta_2^2\theta_3).$$

$$\theta_1^2+\theta_2^2+\theta_3^2=-2(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)=I/2a^2.$$

$$\theta_1^3+\theta_2^3+\theta_3^2=-3(\theta_1^2\theta_2+\theta_1\theta_2^2+\theta_1^2\theta_3+\theta_1\theta_3^2+\theta_2^2\theta_3+\theta_2\theta_3^2)-6\theta_1\theta_2\theta_3=3\theta_1\theta_2\theta_3$$
$$=-3J/4a^3.$$

$$\theta_1^2\theta_2^2+\theta_1^2\theta_3^2+\theta_2^2\theta_3^2=I^2/16a^4.$$

$$\theta_1^3\theta_2^3+\theta_1^3\theta_3^3+\theta_2^3\theta_3^3=-I^3/64a^6+3\theta_1\theta_2\theta_3=(12J^2-I^3)/64a^6.$$

$$\theta_1^2\theta_2+\theta_1\theta_2^2+\theta_1^2\theta_3+\theta_1\theta_3^2+\theta_2^2\theta_3+\theta_2\theta_3^2=-3\theta_1\theta_2\theta_3=3J/4a^3.$$

$$\theta_1^3\theta_2+\theta_1\theta_2^3+\theta_1^3\theta_3+\theta_1\theta_3^3+\theta_2^3\theta_3+\theta_2\theta_3^3=(I/2a^2)(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)=-I^2/8a^4.$$

$$\theta_1^3\theta_2^2+\theta_1^2\theta_2^3+\theta_1^3\theta_3^2+\theta_1^2\theta_3^3+\theta_2^3\theta_3^2+\theta_2^2\theta_3^3=-(\theta_1^2\theta_2^2\theta_3+\theta_1^2\theta_2\theta_3^2+\theta_1\theta_2^2\theta_3^2)$$

$$=-\theta_1\theta_2\theta_3(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)=-IJ/16a^5.$$

$$\therefore A+B+C=\tfrac{1}{4}(80HI-507aJ).$$

$$AB+AC+BC=\frac{1}{64a^2}(342732a^4J^2-28561a^4I^3+179456a^2H^2I^2-262144H^4I$$
$$-27040a^3HIJ-245760aH^3J).$$

$$ABC=\frac{1}{64a^3}(12943360a^3H^3J^2-4826809a^6J^3+33554432H^6J-22970368a^2H^4IJ$$
$$-1142440a^5HIJ^2+3655808a^4H^2I^2J).$$

$$\therefore \frac{a^{12}\Pi}{4096}=-[(2125764a^6J^2+524880a^5HIJ^2-3326427a^6J^2+\tfrac{1}{16}(27761292a^6J^2$$
$$-2313441a^6J^3+14535936a^4H^2I^2-21233664a^2H^4I-2190240a^5HIJ$$
$$-19906560a^3H^3J)+\tfrac{1}{16}(12943360a^3H^3J-4826809a^6J^3+33554432H^6$$
$$-22970368a^2H^4I-1142440a^5HIJ+3655808a^4H^2I^2).$$

$$\therefore a^{12}\Pi=256(2313441a^6I^3-3723875a^6J^2-5065400a^5HIJ+6963200a^3H^3J$$
$$+44204032a^2H^4I-18191744a^4H^2I^2-33554432H^6).$$

GEOMETRY.

306. Proposed by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Find the length of the perpendicular let fall from the point in space (5, 6, 7) upon the line $x=2z-3$, and $y=-3z+1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $\left.\begin{array}{l}x=sz+p\\ y=tz+q\end{array}\right\}$ be the line in space, and m, n, l the point in space. m, n, $l=A$, the vertex of a triangle; p, q, $0=B$, the point where line pierces the xy plane; $\frac{pt-qs}{t}$, 0, $-\frac{q}{t}=C$, the point where line pierces the xz plane.

$$c=AB=\sqrt{[(m-p)^2+(n-q)^2+l^2]}.$$

$$b=AC=\sqrt{\left(\frac{mt-pt+qs}{t}\right)^2+n^2+\left(\frac{lt+q}{t}\right)^2}.$$

$$a=BC=(q/t)\sqrt{(1+s^2+t^2)}.$$

The perpendicular from the vertex A on the side BC is

$$p=\sqrt{c^2-\left(\frac{a^2+c^2-b^2}{2a}\right)^2}.$$

$$\therefore p=\sqrt{(m-p)^2+(n-q)^2+l^2-\left(\frac{(m-p)s+(n-q)t+l}{\sqrt{(1+s^2+t^2)}}\right)^2}$$